Palatini $\beta-$Exponential Inflation with an R^2 Term: Observational Tests

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Introduction

- Inflation is a brief, rapid phase of quasi-exponential expansion in the very early universe, preceding the hot Big Bang [1], solving key puzzles such as the horizon, flatness, monopole, and large-scale structure problems by stretching initial inhomogeneities.
- Supported by precision measurements of Cosmic Microwave Background (CMB) temperature and polarization anisotropies (Planck [2], BICEP/Keck [3]).
- Starobinsky's R^2 model explains inflation via geometric scalaron field and hence, the model does not require a separate, *ad-hoc inflation field* to achieve inflation.

In this poster, we ⋯

- ↪ Adopted the Palatini formalism to derive modified inflationary observables, notably the tensor-to-scalar ratio r, showing distinctive phenomenology compared to the metric formalism.
- ↪ Explored β-exponential inflationary potential in Palatini R^2 gravity, motivated by its emergence from braneworld scenarios and high-energy theories, e.g., *string theory, supergravity*, its tunable parameter β for observational consistency with Planck [2] and BICEP/Keck [3], and its predictive power for next-generation CMB experiments.
- ↪ Examined post-inflation thermal history and reheating dynamics, placing constraints on the reheat temperature and consistency with Big Bang nucleosynthesis (BBN).
- ↪ Presented numerical results for inflationary dynamics and CMB observables.

$\beta-$exponential inflation

- → The β-exponential potential is capable of ending inflation by disrupting the slow-roll regime. As a result, it naturally leads to very small values of the tensor-to-scalar ratio, r [4].
- → In brane cosmology, identifying the radion (the extra–dimension size field) as the inflaton naturally yields the β-exponential potential.

In this work, we study the β-exponential potential model, which was first introduced and analyzed in [4] as a generalization of the phenomenological power-law inflation scenario. The generalized $\beta-$exponential potential is given in the Jordan framework by:

$$V_J(\phi) = V_0 \exp_{1-\beta}(-\lambda\phi) = V_0\left(1-\lambda\beta\frac{\phi}{M_{\rm P}}\right)^{1/\beta}, \tag{1}$$

where the deviation from the pure exponential function is controlled by the constant β, while λ is a dimensionless constant.

Theoretical Framework

We begin by introducing the action in the Jordan framework as:

$$S_J = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{\alpha}{4}R^2 - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V_J(\phi)\right). \tag{2}$$

After applying a Weyl rescaling and a variation with respect the auxiliary field χ the action can be written in the Einstenian framework as

$$S_E \simeq \int d^4x\sqrt{-\tilde{g}}\left(\frac{1}{2}\tilde{R} - \frac{1}{2}\frac{\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi}{\left(1+4\alpha V_J(\phi)\right)} - \frac{V_J(\phi)}{\left(1+4\alpha V_J(\phi)\right)}\right). \tag{3}$$

By using equation (3), we can define the potential model in the Einstein frame as follows:

$$V_E(\phi) = \frac{V_J(\phi)}{\left(1+4\alpha V_J(\phi)\right)} = \boxed{\frac{V_0\left(1-\lambda\beta\phi\right)^{1/\beta}}{\left(1+4\alpha V_0\left(1-\lambda\beta\phi\right)^{1/\beta}\right)}}. \tag{4}$$

Slow-roll approximation

- ⋆ The inflationary parameters, namely, the tensor-to-scalar ratio r, the spectral index n_s, and the running of the spectral index $\frac{dn_s}{d\ln k}$ can be expressed as:

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon, \quad \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\kappa^2,$$

  where $\epsilon = \frac{1}{2}\left(\frac{V_\zeta}{V}\right)^2, \quad \eta = \frac{V_{\zeta\zeta}}{V}, \quad \kappa^2 = \frac{V_\zeta V_{\zeta\zeta\zeta}}{V^2}.$
- ⋆ However the expression of our potential model in terms of $\phi(\zeta)$ is not straightforward and this is why we do our analysis in terms of the original scalar field ϕ.
- ⋆ The number of e-folds N_* in the slow-roll approximation is given by:

$$N_* = \int_{\zeta_e}^{\zeta_*}\frac{V\,\mathrm{d}\zeta}{V_\zeta} \approx 64.7 + \frac{1}{2}\ln\rho_* - \frac{1}{3(1+\omega_r)}\ln\rho_e + \left(\frac{1}{3(1+\omega_r)} - \frac{1}{4}\right)\ln\rho_r, \tag{5}$$

- ⋆ $\rho_e = (3/2)V(\phi_e)$ is the energy density at the end of inflation, ρ_r is the energy density at the end of reheating, $\rho_* \approx V(\phi_*)$ is the energy density when the scale corresponding to k_* exits the horizon, and $\rho_r = \left(\frac{\pi^2}{30}g_*\right)T_{reh}^4$. Here $T_{\rm reh}$ is the reheat temperature; a key parameter for the physics governing the dynamics after inflation, and ω is the equation of the state parameter. The choices: $\omega = 0$ and $\omega = \frac{1}{3}$; have been considered.

Results

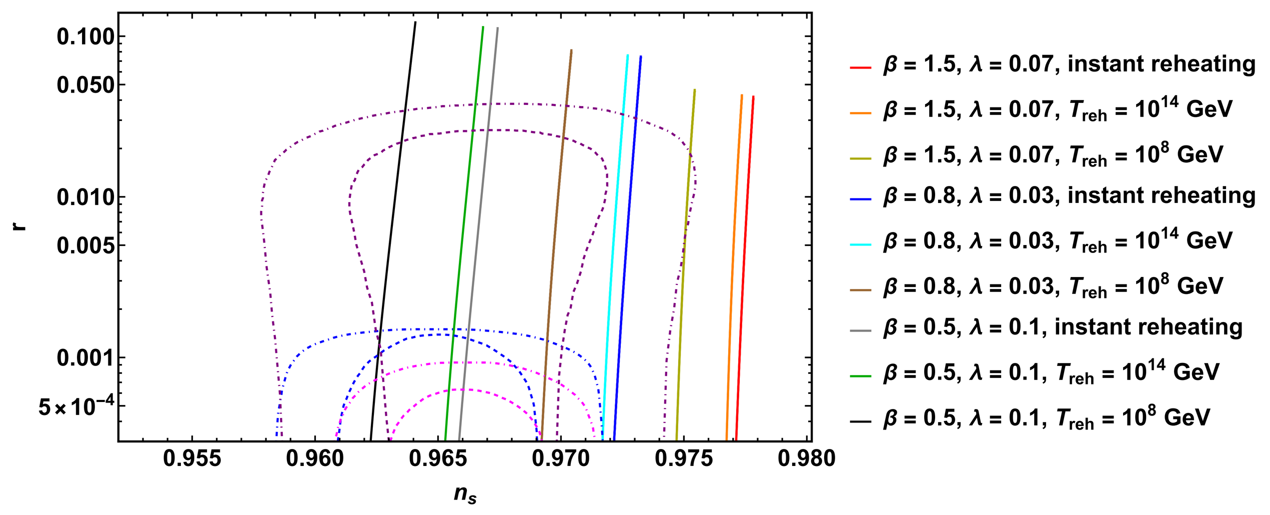

Figure 1: Inflationary predictions (r, n_s) for our model, where α varies within the range $[10^7 - 10^{15}]$. The purple contours (dot-dashed and dashed) represent the recent 95%(68%) confidence levels (CL) from BICEP/Keck [3], while the magenta contours (dot-dashed and dashed) correspond to the prospective constraints from future CMB-S4 observations [5]. The blue contours (dot-dashed and dashed) represent the achievable 95% and 68% CL upper limits from LiteBIRD/Planck in the future [6]

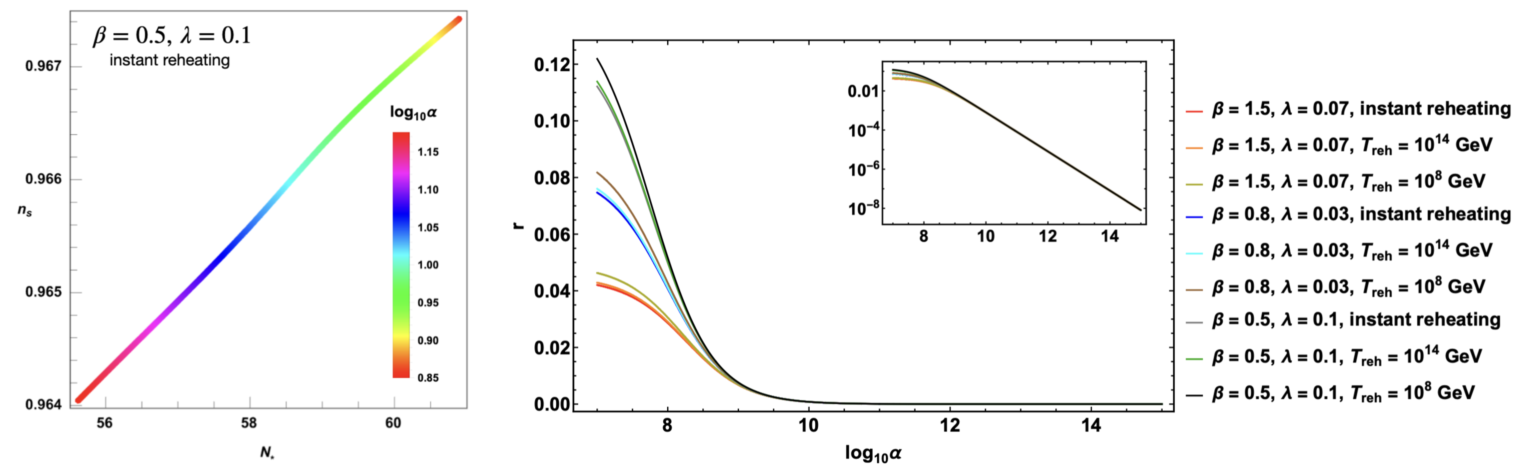

Figure 2: Left panel: shows how the $n_s - N_*$ predictions change depending on the α parameter with $\beta = 0.5$ and $\lambda = 0.1$ for the instant reheating assumption. Right panel: The $\alpha - r$ plane is shown for the selected parameters of our model.

Conclusions

- ↪ Observational viability: The Palatini R^2 β-exponential inflation model aligns tightly with current CMB data (Planck, BICEP/Keck) and remains testable by next-generation experiments (CMB-S4, LiteBIRD).
- ↪ Reheating dynamics matter: Inclusion of reheating temperature ($T_{\rm reh}$) effects refines predictions for n_s and r. Higher $T_{\rm reh}$ favors non-thermal dark matter and leptogenesis, enhancing the model's compatibility with particle cosmology.
- ↪ Effective potential limitations: The β-exponential potential truncates when arguments turn negative, signaling a breakdown of the effective description. This cutoff must guide interpretations of early-universe dynamics.
- ↪ UV imprints: The consistency between theoretical predictions for the $\beta-$exponential inflation model and observational data implies that certain theoretical structures originating from ultraviolet (UV) complete frameworks — such as string theory and quantum gravity — may manifest observable consequences at cosmo- logical scales
- ↪ Unified framework: Our results bridge diverse approaches—connecting braneworld scenarios, modified gravity theories, and conventional inflationary dynamics—thereby enriching the theoretical landscape of primordial inflation.

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