Minimally coupled β-exponential inflation with an R^2 term in the Palatini formulation

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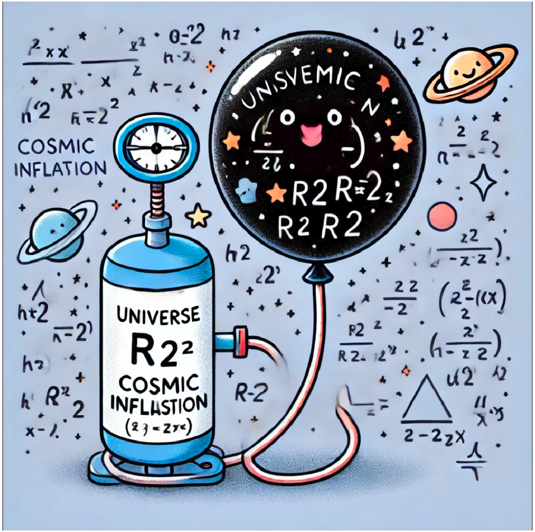
Journal Club

Everything at Once!

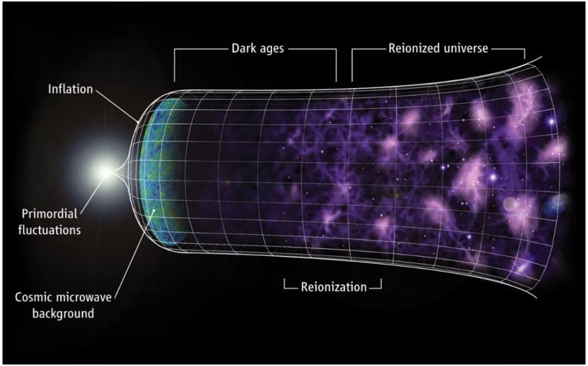
- Inflation
- The Exponential Model
- The Thermal History
- A tailored approach
- Inflationary Observables
- Discussions
- Conclusions



Inflation

"The universe is the ultimate free lunch."

Alan Guth (1947 –)



Inflation

- A period of quasi-exponential cosmic expansion of the universe's size and scale occurred right after the Big Bang singularity.
- Understanding the dynamics of the early universe.
- The recent interest of the scientific community for the past decades [Linde, 2007].
- Solving Cosmological problems, e.g.,
  - The structure problem.
  - Smoothness problem.
  - Flatness problem.
  - Large-scale structure problem, etc.
- Although there are still other open problems, e.g., The small-scale structure problem.
- Supported by measurements from the cosmic microwave background (CMB) anisotropies[Abazajian et al., 2019].

The Exponential Model

- To depict the universe as it expands quasi-exponentially in the early era.
- It is employed to represent the rapid expansion of the universe during the inflationary period, where the scale factor $a(t)$ grows approximately as

$$a(t) \propto e^{Ht}, \tag{1}$$

  and H is the Hubble parameter.
- It helps us in elucidating the observed large-scale homogeneity and isotropy of the universe [Starobinsky, 1980, Guth, 1981].

The Thermal History

- The Coupling between inflaton and the Standard Model (SM) particles is essential in indicating the dynamics of the reheating phase, i.e., the phase of the universe's transition from the inflationary- to a hot and radiation-dominated- era [Bassett et al., 2006].
- These couplings result in the production of SM particles leading to the impact on the thermalization process and the subsequent evolution of the universe.
- The inflaton couples to other fields throughout the reheating phase, converting the remaining energy into new particles that make up the radiation energy density [Kofman et al., 1994].

The Thermal History

- At the end of the reheating phase, thermal equilibrium is reached, and the universe is fully filled with radiation [Baumann, 2022].
- The inflaton's decay produces a particle soup that eventually approaches thermal equilibrium, with the radiation and particle fields present at the time, ensuring that energy is uniformly distributed across the universe's constituents [Mukhanov, 2005], at a certain temperature, *reheat temperature*, T_{reh}, as a result of particle interactions.
- Higher values of reheat temperatures are favorable for non-thermal dark matter production and leptogenesis.
- A comprehensive analysis considering the reheating scenarios is ahead!

A tailored approach

The $\mathbf{R}^2$ term

- It can be derived for the inflationary expansion by introducing a scalar degree of freedom known as the scalaron, which acts as a role of the inflaton; hence, the model does not require a separate, ad-hoc inflation field to achieve inflation, making it one of the earliest and most accomplished inflationary models [Starobinsky, 1980].
- The inclusion of the term αR^2 can drive inflation and improve the ultraviolet (UV) behavior of the theory of gravity. Therefore assuring a well-motivated starting point for the physics analysis at very high energies [Tenkanen, 2019].
- We will see its results on the computations across the discussions.

A tailored approach

The Palatini formulation

- It is defined as an independent variation with respect to the metric, an independent connection, and a reduced standard deviation.
- Theories based on this formalism satisfy the metric postulates [Will, 2018].
- It predicts the inflationary observables, especially for the tensor-to-scalar ratio (r) which makes it more prevailing than the metric formulation [Bauer and Demir, 2008, Bostan, 2021].
- It can potentially offer better alignment with the measurements from CMB anisotropies and large-scale structure surveys.[Koivisto and Kurki-Suonio, 2006, Borunda et al., 2008, Gialamas et al., 2023], and hence it provides us with a robust and comprehensive analysis of the inflationary dynamics.

A tailored approach

The β-exponential model

- It is well-studied and aligned with the current cosmological data through the literature
[Alcaniz and Carvalho, 2007, Santos et al., 2018, dos Santos et al., 2022].
- The model can appear in the framework of brane cosmology,
[Dvali and Tye, 1999], in which the inflaton is regarded as the field representing the size of extra-dimension, which is strongly motivated for primordial inflation.
- It can fulfill the disruption of the slow-roll regime with the end of inflation [Santos et al., 2018], thus it makes the tiny values for the tensor-to-scalar ratio (r) [Alcaniz and Carvalho, 2007].
- We consider the following potential [Alcaniz and Carvalho, 2007]:

$$V(\phi) = M^4 \exp_{1-\beta}\left(-\lambda\phi/M_{\rm P}\right) , \tag{2}$$

A tailored approach

The β-exponential model

where the definition of the generalized exponential function $\exp_{1-\beta}$ is as follows:

$$\exp_{1-\beta}(f) = [1 + \beta f]^{1/\beta} \ , \tag{3}$$

for $\begin{cases} 1 + \beta f > 0 \\ \exp_{1-\beta}(f) = 0, \text{ otherwise.} \end{cases}$

A tailored approach

The β-exponential model

- The Jordan frame potential, $V(\phi)$, for the β-exponential inflation can be written in the following form [Lillepalu and Racioppi, 2023]:

$$V(\phi) = V_0 \left(1 - \lambda\beta \frac{\phi}{M_{\rm P}}\right)^{1/\beta} , \tag{4}$$

where the deviation from the pure exponential function is controlled by constant β, while λ is a dimensionless constant.

- To derive the potential expression in the Einstein framework, we start by introducing the following action:

$$S_J = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_{\rm P}^2 R + \frac{\alpha}{4}R^2 - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\right) , \tag{5}$$

A tailored approach

The β-exponential model

- After applying Weyl rescaling and making the following field redefinition:

$$\mathrm{d}\zeta = \frac{\mathrm{d}\phi}{\sqrt{1+\frac{4\alpha}{M_{\mathrm{P}}^4}V(\phi)}} = \frac{\mathrm{d}\phi}{\sqrt{Z(\phi)}}, \tag{6}$$

- We get the following action:

$$S_E \simeq \int d^4x\sqrt{-\tilde{g}}\left(\frac{1}{2}M_{\mathrm{P}}^2\tilde{R} - \frac{1}{2}\frac{\nabla^\mu\phi\nabla_\mu\phi}{\left(1+\frac{4\alpha}{M_{\mathrm{P}}^4}V(\phi)\right)} - \frac{V(\phi)}{\left(1+\frac{4\alpha}{M_{\mathrm{P}}^4}V(\phi)\right)}\right). \tag{7}$$

- Now we can indicate the Einstein frame potential from the action as follows:

$$V_E(\phi) = \frac{V(\phi)}{\left(1+\frac{4\alpha}{M_{\mathrm{P}}^4}V(\phi)\right)}. \tag{8}$$

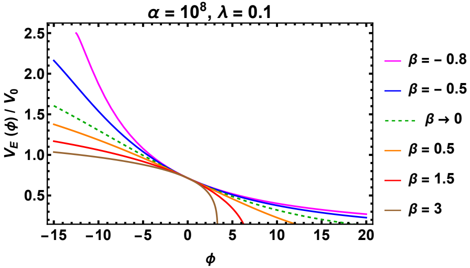
A consequence of the R^2 term.

A tailored approach

The β-exponential model

- We expressed our potential in the Jordan framework as in Eq.(4), so our final expression for the potential in the Einstein framework can be given as:

$$V_E(\phi) = \frac{V_0\left(1-\lambda\beta\frac{\phi}{M_{\rm P}}\right)^{1/\beta}}{\left(1+4\alpha\frac{V_0\left(1-\lambda\beta\frac{\phi}{M_{\rm P}}\right)^{1/\beta}}{M_{\rm P}^4}\right)}. \quad (9)$$



Inflationary Observables

- Recently more precise constraints on the inflationary predictions have been provided by BICEP/Kick [Ade et al., 2021] which tightens the tensor-to-scalar ratio (r) to $r < 0.035$ at 95% CL.
- It also constraint the spectral index (n_s) to the range $[0.957 - 0.976]$ at 2σ CL.
- CMB-S4 survey is aiming for $r \simeq \mathcal{O}(10^{-3})$.
- It is expected that LiteBIRD [Allys et al., 2022] experiment will be able to test the inflationary models more precisely.

Inflationary Observables

The slow-roll parameters

- We begin by introducing the slow-roll parameters in terms of ϕ under the following considerations:

  The subscripts "*" denotes the quantities when the pivot scale exits the horizon, and "e" refers to the end of inflation.
  Here x refers to:

$$x \equiv 1 - \beta\lambda\phi_* . \tag{10}$$

  We take into consideration the slow-roll condition to calculate ϕ_e by using

$$\epsilon(\phi_e) \simeq 1. \tag{11}$$

  - For our following computations the expressions are introduced in terms of the scalar field ϕ to calculate the the inflationary observables more precisely due to the complex structure of the canonical scalar field ζ .

Inflationary Observables

The slow-roll parameters

$$\epsilon_\phi = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad \eta_\phi = \frac{V''}{V} \quad , \quad \kappa_\phi = \frac{V'V'''}{V^2}. \tag{12}$$

$$\epsilon = Z\epsilon_\phi \quad \Rightarrow \quad \epsilon(\phi_*) \simeq \frac{\lambda^2}{8\alpha V_0 x^{\frac{1}{\beta}+2} + 2x^2}, \tag{13}$$

measures the steepness of the inflationary potential.

$$\eta = Z\eta_\phi + \mathrm{sgn}\left(V'\right) Z'\sqrt{\frac{\epsilon_\phi}{2}} \quad \Rightarrow \quad \eta(\phi_*) \simeq \frac{\lambda^2\left(-2\beta + \frac{3}{4\alpha V_0 x^{1/\beta}+1} - 1\right)}{2x^2}, \tag{14}$$

describes the slope of the potential changes that affect the stability and duration of inflation.

Inflationary Observables

The slow-roll parameters

$$\kappa^2 = Z\left(Z\kappa_\phi^2 + 3\mathrm{sgn}(V')Z'\eta_\phi\sqrt{\frac{\epsilon_\phi}{2}} + Z''\epsilon_\phi\right), \quad \text{and} \quad \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\kappa^2 \,. \tag{15}$$

investigates finer details about the shape of potential and the dynamics of inflation.

- However the running of the spectral index $\left(\frac{dn_s}{d\ln k}\right)$ values are too small for our model, as we will show in our results.
- The spectral index:

$$n_s = 1 - 6\epsilon + 2\eta \quad \Rightarrow \quad n_s(\phi_*) \simeq 1 - \frac{(2\beta+1)\lambda^2}{x^2}. \tag{16}$$

- The tensor-to-scalar ratio:

$$r = 16\epsilon \quad \Rightarrow \quad r(\phi_*) \simeq \frac{8\lambda^2}{x^2\left(4\alpha V_0 x^{1/\beta} + 1\right)}. \tag{17}$$

dependency on α!

Inflationary Observables

- The amplitude of the curvature perturbation can be given by:

$$\Delta_{\mathcal{R}} = \frac{1}{2\sqrt{3}\pi}\frac{V^{3/2}}{\sqrt{Z}|V'|} \quad \Rightarrow \quad \Delta_{\mathcal{R}}^2(\phi_*) \simeq \frac{V_0^3 x^{3/\beta}}{12\pi^2\left(4\alpha V_0 x^{1/\beta}+1\right)^4\left|\frac{x^{\frac{1}{\beta}-1}\lambda V_0}{(4\alpha V_0 x^{1/\beta}+1)^2}\right|^2}. \tag{18}$$

- From the recent Planck measurements [et al., 2020] the value of the pivot scale can be taken as $k_* = 0.002\,\text{Mpc}^{-1}$ which results in:

$$\Delta_{\mathcal{R}}^2 \approx 2.1 \times 10^{-9}. \tag{19}$$

Inflationary Observables

- The number of e-folds N_* in the slow-roll approximation is given by:

$$N_* = \mathrm{sgn}(\mathrm{V}') \int_{\phi_\mathrm{e}}^{\phi_*} \frac{\mathrm{d}\phi}{\mathrm{Z}(\phi)\sqrt{2\epsilon_\phi}} \simeq \frac{\phi_*(\beta\lambda\phi_* - 2)}{2\lambda}, \tag{20}$$

supposing a standard thermal history for the pivot scale $k_* = 0.002\,\mathrm{Mpc}^{-1}$ we can write N_* as [Liddle and Leach, 2003]:

$$N_* \approx 64.7 + \frac{1}{2}\ln\frac{\rho_*}{M_\mathrm{P}^4} - \frac{1}{3(1+\omega_r)}\ln\frac{\rho_e}{M_\mathrm{P}^4} + \left(\frac{1}{3(1+\omega_r)} - \frac{1}{4}\right)\ln\frac{\rho_r}{M_\mathrm{P}^4}, \tag{21}$$

Inflationary Observables

where

- ρ_* is the energy density when the scale corresponding to k_* exits the horizon.
- ρ_e is the energy density at the end of inflation.
- ρ_r is the energy density at the end of reheating; responsible for the *reheat temperature.*
- They are given by:

$$\rho_* = \frac{3\pi^2 \Delta_{\mathcal{R}}^2 r}{2}, \quad \rho_e = (3/2)V(\phi_e), \quad \rho_r = \left(\frac{\pi^2}{30} g_*\right) T_{reh}^4, \tag{22}$$

here the standard model value $g_* = 106.75$, gives the number of relativistic degrees of freedom, can be employed to compute ρ_r, and T_{reh} indicates the reheat temperature.

- ω_r is the equation of the state parameter during reheating.

Inflationary Observables

- In this work we consider two scenarios for (N_*):
    1. $\omega_r = 1/3$; *instant reheating* which reduces the expression to:

$$N_* \approx 64.7 + \frac{1}{2}\ln\frac{\rho_*}{M_{\rm P}^4} - \frac{1}{4}\ln\frac{\rho_e}{M_{\rm P}^4}. \tag{23}$$

       doesn't depend on the T_{reh}.
    2. $\omega_r = 0$ which reduces the expression into:

$$N_* \approx 64.7 + \frac{1}{2}\ln\frac{\rho_*}{M_{\rm P}^4} - \frac{1}{3}\ln\frac{\rho_e}{M_{\rm P}^4} + \frac{1}{12}\ln\frac{\rho_r}{M_{\rm P}^4}, \tag{24}$$

       here we can see the dependency on the T_{reh} hence effect on the inflationary predictions.
       We take a range of T_{reh} as $[10^8 - 10^{14}]$ GeV.

Discussions

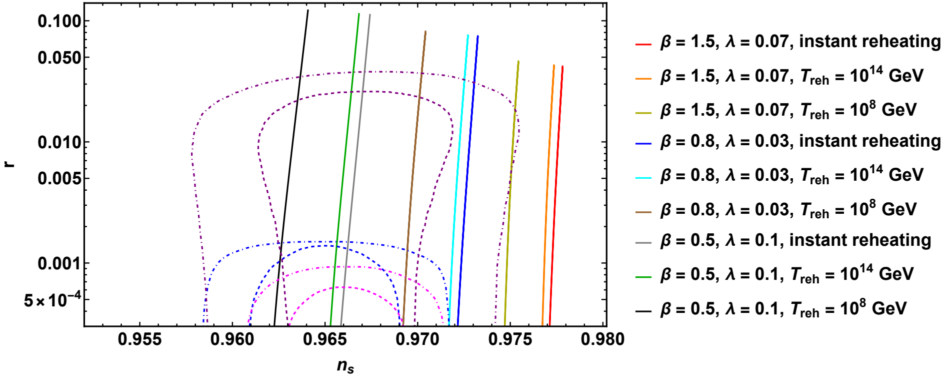
N_*	$n_s(\phi_*)$	$r(\phi_*)$	$\mathrm{d}n_s/\mathrm{d}\ln k\ (\phi_*)$	ϕ_*	ϕ_e	V_0
			$\alpha = 10^8,\ \lambda = 0.1,\ \beta = 0.5$			
61	0.967	0.049	-5.29×10^{-4}	35.68	21.40	6.58×10^{-9}
55	0.964	0.051	-6.46×10^{-4}	34.91	21.40	8.04×10^{-9}
45	0.956	0.055	-9.64×10^{-4}	33.49	21.39	1.20×10^{-8}
			$\alpha = 10^{15},\ \lambda = 0.1,\ \beta = 0.5$			
61	0.967	8.04×10^{-9}	-5.34×10^{-4}	35.64	20.07	6.65×10^{-9}
55	0.964	8.04×10^{-9}	-6.47×10^{-4}	34.91	20.07	8.05×10^{-9}
45	0.956	8.04×10^{-9}	-9.64×10^{-4}	33.49	20.06	1.20×10^{-8}

Table: Results of the minimally coupled β-exponential model parameters.

Inflationary Observables

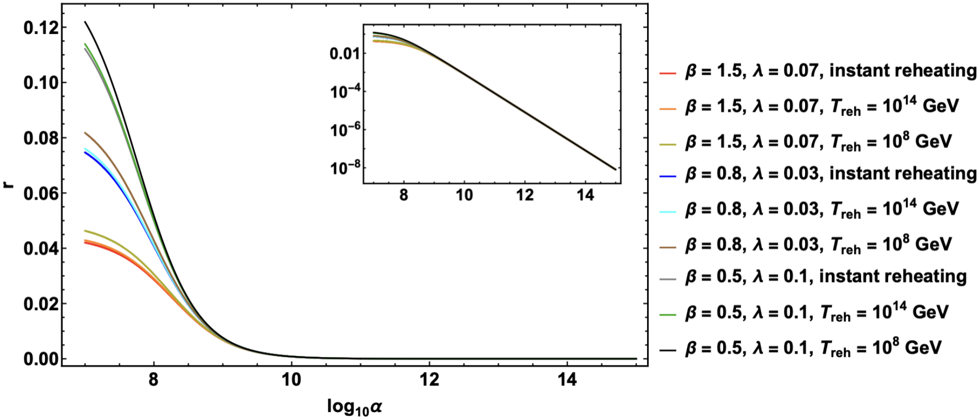
The slow-roll parameters

- By varying α in range of $[10^7 - 10^{15}]$, and including the BICEP/Keck [Ade et al., 2021] with Purple, CMB-S4 [Abazajian et al., 2019] with magenta, and LiteBIRD/Planck constraints [Allys et al., 2022] with blue.



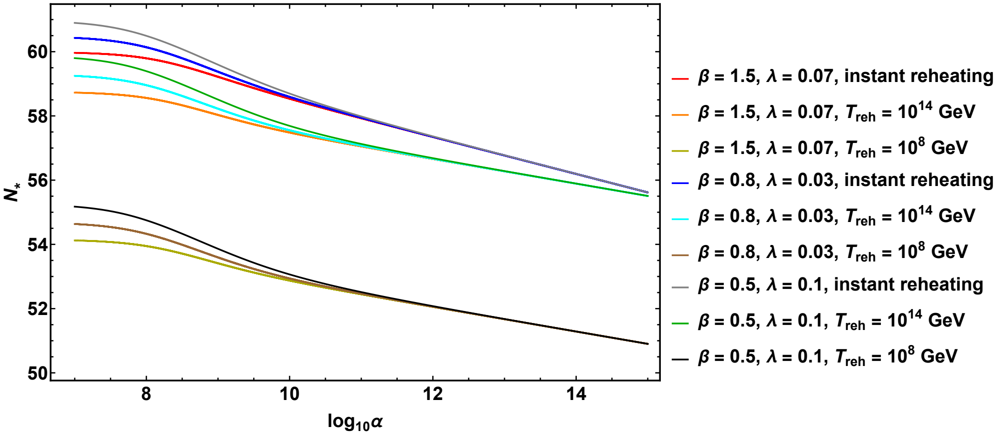
Discussions

- The dependency on the α, λ, and T_{reh}.



Discussions

- Higher α inversely proportional with N_* $\Rightarrow$ a stronger R^2 term reduces the duration of inflation.



Conclusions

- Aligning the inflationary predictions with various cosmological data and possible future sensitivities.
- Reaching more precise inflationary observables.
- Investigating different choices of reheat temperature scenarios that have direct consequences on the inflationary predictions.
- Contributing to the build a more complete picture of the current work regarding inflation.

The work is available on the arXiv: **2409.10398**.

Feel free to contact me for any discussions related to this work:
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Acknowledgments

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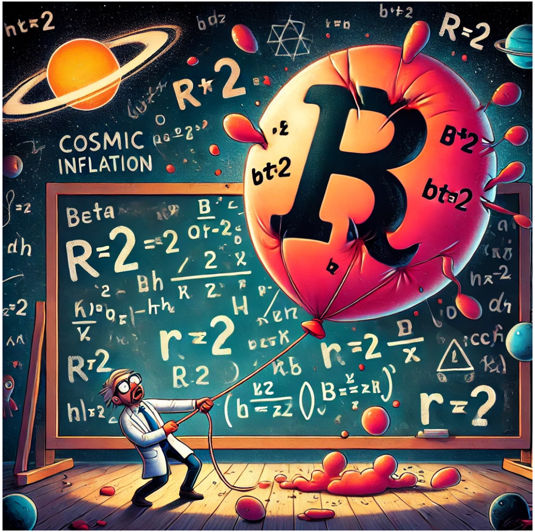
- `Open AI` for the 2 images in the second and last slide.
- `Google images` for the inflation-related picture.

Thank You!



Backup Slides!

Inflationary observables

- We can find (n_s), (r), and $(\Delta_{\mathcal{R}})$ in terms of the number of efolds (N_*) which helps us to have a better understanding of the inflationary dynamics.
- We consider two different approximations:
  - The case of $\beta\phi_*^2/2 \gg \phi_*/\lambda \quad \Rightarrow \quad \left(x \approx 1 \mp (\lambda\sqrt{2\beta\, N_*}\right)$
    Resulting in:

$$n_s \simeq 1 - \frac{(2\beta+1)\lambda^2}{\left(1 \mp (\lambda\sqrt{2\beta N_*})\right)^2},$$

$$r \simeq \frac{8\lambda^2}{\left(1 \mp (\lambda\sqrt{2\beta N_*})\right)^2 \left(4\alpha V_0 \left(1 \mp (\lambda\sqrt{2\beta N_*})\right)^{1/\beta} + 1\right)}, \tag{25}$$

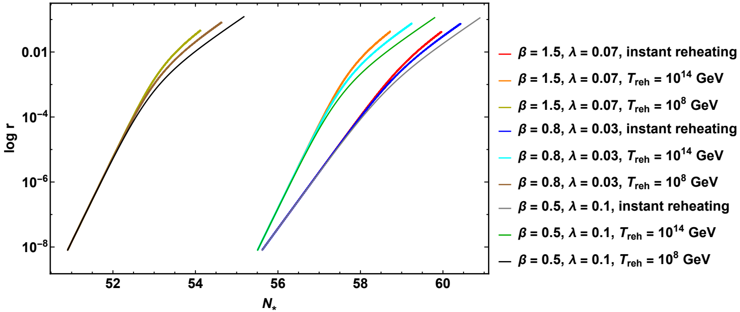
$$\Delta_{\mathcal{R}}^2 \simeq \frac{V_0^3 (1 \mp (\lambda\sqrt{2\beta N_*}))^{3/\beta}}{12\pi^2 \left(4\alpha V_0 (1 \mp (\lambda\sqrt{2\beta N_*}))^{1/\beta} + 1\right)^4 \left| \frac{(1\mp(\lambda\sqrt{2\beta N_*}))^{\frac{1}{\beta}-1} \lambda V_0}{\left(4\alpha V_0 (1\mp(\lambda\sqrt{2\beta N_*}))^{1/\beta}+1\right)^2} \right|^2}. \tag{26}$$

Inflationary Observables

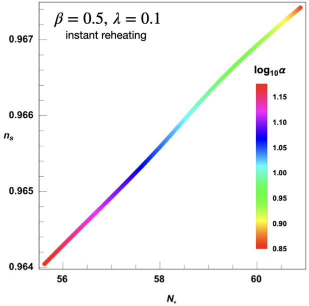
- The case of $\beta\lambda\phi_* \ll 1 \quad \Rightarrow \quad x \approx 1 + \lambda^2\beta\, N_*$
  resulting in:

$$n_s \simeq 1 - \frac{(2\beta+1)\lambda^2}{(1+\lambda^2\beta N_*)^2},$$
$$r \simeq \frac{8\lambda^2}{(1+\lambda^2\beta N_*)^2\left(4\alpha V_0(1+\lambda^2\beta N_*)^{1/\beta}+1\right)}. \quad (27)$$

$$\Delta^2_{\mathcal{R}} \simeq \frac{V_0^3(1+\lambda^2\beta N_*)^{3/\beta}}{12\pi^2\left(4\alpha V_0(1+\lambda^2\beta N_*)^{1/\beta}+1\right)^4\left|\frac{(1+\lambda^2\beta N_*)^{\frac{1}{\beta}-1}\lambda V_0}{\left(4\alpha V_0(1+\lambda^2\beta N_*)^{1/\beta}+1\right)^2}\right|^2}. \quad (28)$$



0.01
10^{-4}
10^{-6}
10^{-8}
log r
52
54
56
58
60
N_*
β = 1.5, λ = 0.07, instant reheating
β = 1.5, λ = 0.07, $T_{reh} = 10^{14}$ GeV
β = 1.5, λ = 0.07, $T_{reh} = 10^{8}$ GeV
β = 0.8, λ = 0.03, instant reheating
β = 0.8, λ = 0.03, $T_{reh} = 10^{14}$ GeV
β = 0.8, λ = 0.03, $T_{reh} = 10^{8}$ GeV
β = 0.5, λ = 0.1, instant reheating
β = 0.5, λ = 0.1, $T_{reh} = 10^{14}$ GeV
β = 0.5, λ = 0.1, $T_{reh} = 10^{8}$ GeV



$\beta = 0.5,\ \lambda = 0.1$
instant reheating
$\log_{10}\alpha$
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.967
0.966
0.965
0.964
n_s
56
58
60
N_*